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RESEARCH ARTICLE

Fractal description of dendrite growth during electrochemical migration

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Abstract

In this paper, the usability of fractal description of dendrites in failure analysis is discussed. Dendrites grow during electrochemical migration (ECM) causing failures in the form of short circuits between conductive wirings and leads in microcircuit interconnection and packaging elements, especially on printed wiring boards. The parameters of the dendrite growth process are influenced by functional (electrical) and environmental conditions as well as by the material composition of metallisation stripes and surface finishes. Mathematically, dendrites can be described as fractals and therefore they can be characterised with fractal dimension values calculated by various computerised pattern processing methods. We have found relationship between chemical composition of surface finishes and fractal dimensions of dendrites grown from them. This hypothesis has been proved in three steps: first, dendrites were grown by simple water drop test (WDT) series in order to get good quality, reproducible test specimens; in the second step, optical photomicrographs were taken from the dendrite structures and finally these micrographs were analysed using computerised algorithms. The result is that fractal dimensions are material specific and so they can be used to describe the material composition. This is not only an important theoretical result in understanding dendrite growth mechanisms, but it also has a useful practical aspect: short detection and optical inspection can be combined with simple calculations to identify materials involved into electrochemical migration failures without making more complicated elemental chemical analysis.

Keywords

electrochemical migration · water drop test · applied fractal theory.

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1 Introduction

ECM is an important and entrancing but unfortunately frequently misinterpreted and not too widely-known psychical-chemical failure mechanism limiting the realization of fine-pitch structures in electronics technology. Manufacturing technology of printed wiring boards and assembly technology of printed circuit boards are among the highly threatened processes because this phenomenon can cause shorts in form of dendrite structure between closely-spaced unisolated wirings, leads of mounded devices. The entire theoretical model was defined by G. Harsányi who has given a comprehensive view in his dissertation for the first time [1]. The essence of ECM is the material transport which can occur in solid states, in fusions and in solutions. We have made a study of the latst one that can occur if there is thin moisture-film between unisolated wirings or leads and is organic or inorganic impurity in it. A very special case was observed after reflow soldering using no-clean flux where the impurity was succinate-acid. This is only one of the unsolved industrial problems that prove the actuality of this topic [2].

Dendrites are treatable as fractal phenomena because these special formations are in accordance with the most significant criteria of fractal theory which is proved in many papers on this topic [3]. Correlation has been observed between material composition of surface finishes, morphology of dendrites and kinetic of dendrites' growth. Chemical composition of dendrites can be examined with scanning electron microscopy using energy dispersive spectrum analysis. Kinetic behaviour can be described by mean time to failure (MTTF) which can be predicted from standardized accelerated life time test methods (highly accelerated stress test, steady state temperature-humidity-bias test and so on) and the results of simple water drop test series give also useful information [4]. MTTF can be characterized with material composition being in relation to fractal dimension of dendrites and if the result of failure analysis is a short caused by ECM then it is worth examining the shapes and forms, structures of dendrites and the incidentally arising precipitates [5].

Shorts can be detected with in-circuit tests and the so called automatic optical inspection is also generally applied in failure analyses. These techniques are widely held in today's assembly technology and their costs will be cheaper day by day. According to our conclusion the cause of failure can be found with these equipments, can be eliminated with simple and cheap methods and the most efficient protection can be used. This can be done for example by using another surface finish, more effective cleaning method, environment-friendly conformal coating or changing on solder material – flux type combination.

The ECM is such a specialized subject that the researcher must have sufficient practical experience and has to fully understand this special failure phenomenon. The material composition of dendrites (and so the material composition of surface finishes from which they are grown) can be determined with adequate precision by the use of some good quality optical photomicrographs, without using any expensive examination. Besides material composition the potential reason for failures and their simplest and most efficient mode of elimination can be also established in the same way. This depends significantly on subjective factors and this is why we have made attempts to work out an objective examination method that can be useful for engineers and scientists engaged in this field. One potential method is the simple and brief description of dendrite's morphology and their manner of growth. The efficiency of aforementioned method can be increased with the help of our new revelation which will be discussed particularly.

2 Experimental

2.1 Mathematical background [6]

The growth of dendrites happens fractal-wise. The term fractal was coined in 1975 by Mandelbrot from the Latin fractus, meaning "broken" or "fractured". In colloquial usage, fractal is a shape that is recursively constructed or self-similar, that is, a shape that appears similar at all scale of magnification and is referred to as "infinitely complex". A self-similar object is exactly or approximately similar to a part of itself, the whole has the same shape as one or more of the parts. Many objects in the real world, such as coastlines, are statistically self-similar: parts of them show the same statistical properties at many scales.

Self-similarity is a typical property of fractals, and the dendrites have this condition. Dendrites grown from different kind of material can be well differentiated by the help of their fractal dimension. Mathematical fractals such as the Koch Curve are exactly self-similar over an infinite range of scales. A dendrite as a natural phenomenon is statistically self-similar over a large but finite range of scales and is better suited to a fractal rather than a Euclidean description. One relatively easy method of computing the fractal dimension is the box counting algorithm. If we have a set A in the plane, we first place a grid of mesh size r over the region of the plane involved. Then we count the number of boxes of width r contain a piece A, which we denote $Nr(A)$. Then take the limit

$$D = \lim_{r \to 0} \frac{\log N_r(A)}{-\log r} \tag{1}$$

This algorithm is easily implemented on a computer, which can relatively easily count the boxes which contain part of an image. Most of the programs which were applied in our experimentation use this algorithm, so we have to introduce its basic numerical steps. The specific example is the Koch curve which can be seen on Fig. 1. In the first step (Fig. 2) the side of the grid is $r = 1$, in the second step (Fig. 3) the side of the grid is $r = 0.5$, in the third step (Fig. 4) the side of the grid is $r = 0.25$ and so on. The program carries on this simply method as long as the mesh size hits the lower limit. The lower limit is the pixel size. The results are illustrated on logarithmic scaled diagram. On the y-axis can be seen the logarithm of the number of the boxes, on the x-axes are represented the negative logarithmic values of the mesh sizes (Fig. 5). The curve is approximately a line and the negative slope of this line is the fractal dimension defined in Eq. (1).

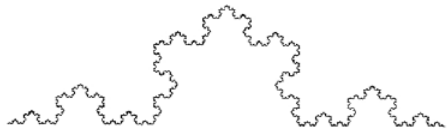


Fig. 1. The Koch Curve

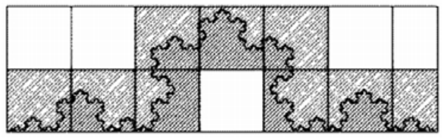


Fig. 2. First iteration, r = 1 and N (r) = 9.

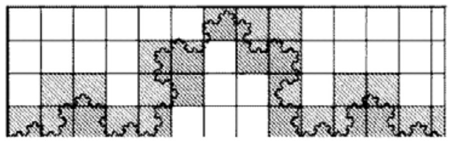


Fig. 3. Second iteration, r = 0.5 and N(r) = 25.

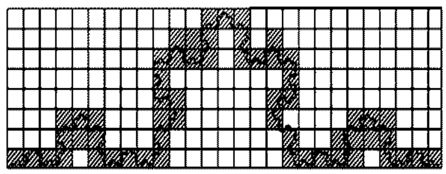


Fig. 4. Third iteration, $r = 0.25$ and $N(r) = 58$.

There is standardized test board to carry out examinations to study the ECM but we have applied simple parallel arranged pair of electrode on glassfiber-epoxy (FR4) substrate. The growth of dendrites was observed by optical microscopy and, if it was necessary, the test voltage was adjusted. After formation of electrical shorts the test potential was decreased and held at 0.5V during evaporation of the water droplet. The time of short formation can be simply and precisely measured using a cascaded resistor. Because dendrites of the various materials grow on the top of the water drop, the dendrites are preserved by applying the

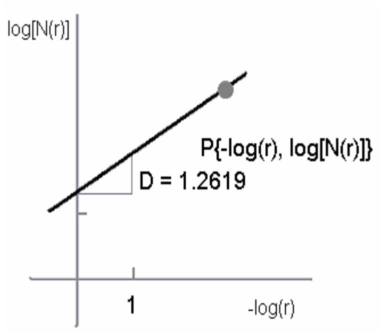


Fig. 5. Dimension of Koch Curve with box-counting algorithm (theoretical value).

low voltage until the water evaporates. If we turn off the power supply in these cases, the evolved structure will collapse. So, the test specimens can be easily moved without damaging preserved dendrite structures and with adequate storage they can be kept for a long time. Therefore there is no need to take optical photomicrographs immediately after the WDT growths thus the photomicrographs can be taken elsewhere.

Test specimens were backlit for photomicrography in order to separate any disturbing impurity and arisen precipitate. The latter is important but in our case the frame of dendrite structure is the examined property. The values of magnification were not uniform advised, the reproducibility was not relevant because dendrites as fractals are self-affine objects and so the magnification does not change the values of fractal dimensions. Photos were saved in colour bitmap (BMP) image format, and 1024 x 768 resolutions were chosen for picture definition which was never changed. Two image processings were applied, such as decreasing colour depth to 1 and 8 bit per pixel (BPP), and so the results were black and white images and grayscale images. Two examples can be seen in Fig. 6.

Other image processing was not applied, even if the evolved dendrite patterns were not perfect. Freeware picture imaging software was used which can be found on the WEB. After filtering images, the file size of pictures was so small that the process time of our mathematical algorithms was very fast. They are introduced in subsection 2.3.

2.2 The applied softwares and their usability

Five different algorithms were used to determine the fractal dimensions of the images. Dff.m and Dsz.m programs were written in text format by us on the basis of A. K. Demcu's Fraktdim.m MATLAB file. Practically functional principles were in both cases in agreement with the original program but there were two important differences which must be brought to reader's notice. In order to decrease the complexity of the algorithm, other functions and procedures were applied so that the running time of software was significantly decreased as well. This is not a negligible issue if the resolution of pictures is too large or the computer is relatively slow and has not enough data memory. The other difference is that Dff.m program analyses only black and white images and does not decrease the colour depth. The colour conversion was executed by using picture editor before running Dff.m file. Both Dsz.m and Fraktdim.m calculates the same type of fractal dimension of a grayscale image, and they give the same result but the computation of Dsz.m is essentially faster.

The fft2d.m file can be free downloaded from the public website of MathWorks Corporation. This is a computational method which is applied in geodesy to analyze photos (images) taken from the relief of an examined area, using FFT (Fast Fourier Transformation) algorithm [8]. Fourth procedure is the freeware FracLab toolbox of MATLAB program package. The ImageJ – FracLac.jar (the fifth algorithm) has an easily adjustable graphical user interface and does not require the installation of MATLAB.

We have selected the Sierpinski Gasket (one of the well-known mathematical fractal) because we would like to demonstrate that the applied softwares are suited for calculating fractal dimensions. The Sierpinski gasket can be seen on Fig. 7. The theoretical dimension of this object is $\log(3)/\log(2) \sim 1.5850$. The with Dff.m program calculated value can be seen on Fig. 8, and the results of the five software can be seen in Tab. 1.

Because the computed and theoretical fractal dimensions are nearly equal, these programs can be used for calculating the fractal dimensions of our processed images made from dendrites grown from different kinds of surface finishes.

Tab. 1. Results of the five softwares in the case of Sierpinski Triangle.

Dff.m	Dsz.m	fft2d.m	FracLab	FracLac.jar
1.5886	1.5864	1.5993	1.5966	1.6024

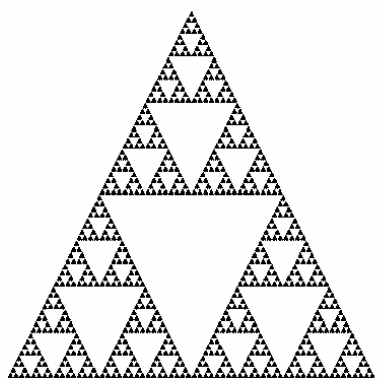


Fig. 7. The Sierpinski Gasket from triangles.

2.3 Water drop test series

Three experiment series were carried out in our research whose primary goal was the examination of fractal dimensions of dendrites' grown from different kinds of surface finish using the WDT. Preparation of test specimens was always made according to the method introduced in subsection 2.2. Fractal dimensions of four kinds of material were examined at the first

Black and white image.

Grayscale image.

Fig. 6. Photomicrographs of dendrite grown from immersion silver surface finish after image processing

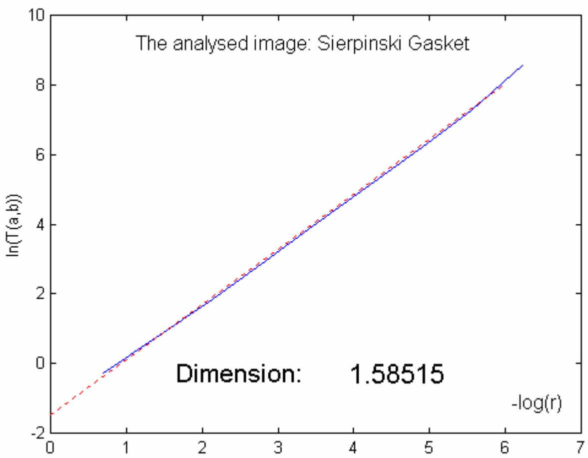


Fig. 8. Calculated dimension value for the Sierpinski Gasket.

experiment series. They were the following: electroplated tin, immersion silver, pure copper and SAC387 solder alloy. Abbreviation "SAC387" means that the compounds are 95,5 % tin, 3,8 % copper and 0,7 % silver.

Materials (surface finishes) examined at the second experiment series were pure lead, pure tin and eutectic tin-lead solder alloy. In these cases the technology of metallization was not electroplating and immersion, but Hot Air Solder Leveling (HASL). In the third experiment series the coatings were formed by pure lead, pure tin and tin-lead solders (Sn80 %Pb20 %, Sn63 %Pb37 % and Sn20 %Pb80 %) also by HASL technology. Sixty test samples were prepared in all metallization. We examined eight kinds of material so the totalized sum of test specimens are 480 samples. The five software programs, which were introduced in subsection 2.2, were applied to analyze the optical photomicrographs taken from test specimens of every experiment series.

3 Results and Discussion

At first we have had to analyze the distribution of data calculated by the above-mentioned software. The "NORMPLOT" function of MATLAB has been applied to get information from probability distribution. Gaussian was observed, two examples can be seen on Figs. 9-10. The most important statistical parameters of fractal dimensions are mean value and deviation which were evaluated using tables and diagrams. It is difficult to draw distinctions between results because the difference of dimension values is not sufficiently large. They are also over 1 and below 2 in 2D but the scale of deviation from each other are insignificant.

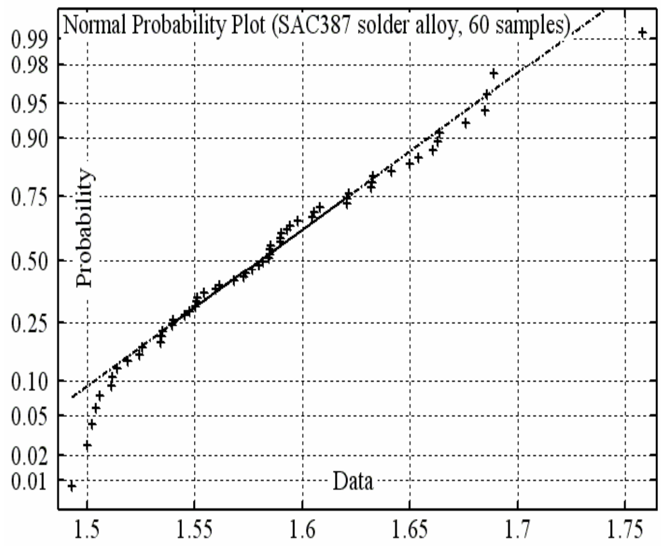


Fig. 9. Normal Probability Plot (SAC387 solder alloy, 60 samples)

In order to have a more clearly representation, mean values of fractal dimensions (S) and their deviations (σ) were transformed: $S' = (S-c) \times 1000$ and $\sigma' = \sigma \times 1000 (1 \leq c \leq 2)$. By illustrating S' and σ' instead of S and σ, it is easier to show difference between values visually, as illustrated in the diagrams. This method is not the best solution because the diagrams do not show the characteristics of the whole distributions. An example is shown in Fig. 11.

Combining the equation of the normal distribution with histograms:

$$y = \max\{\text{hist (Me,c)}\} \cdot \exp(-\frac{1}{2} \cdot \frac{(x-m)^2}{s^2}) \qquad (2)$$

where m means mean value and s means standard deviation. The hist(Me,c) function puts the elements of Me into c equally

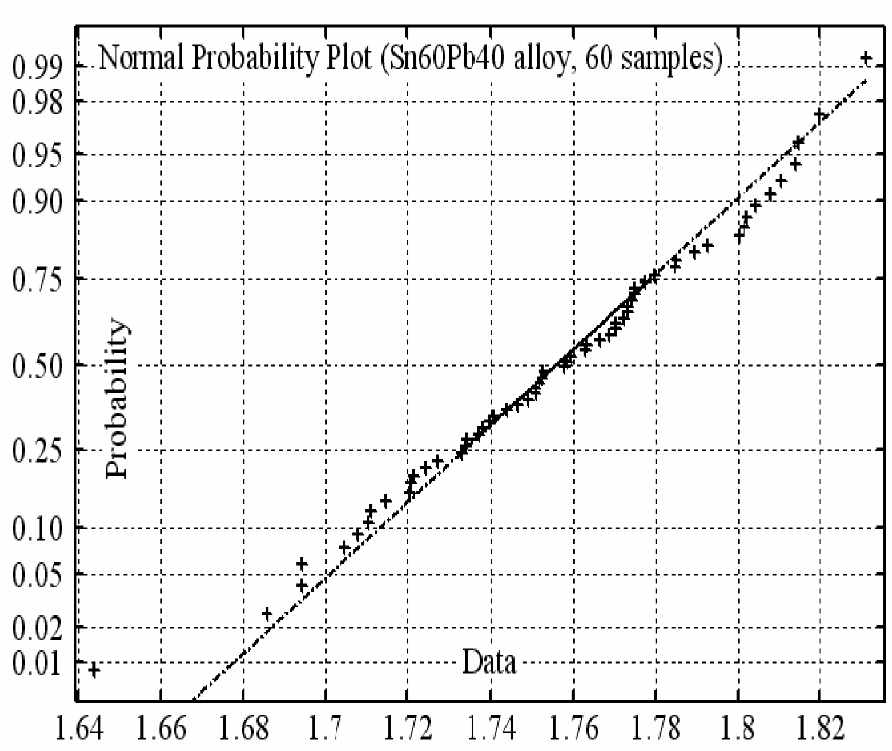


Fig. 10. Normal Probability Plot (Sn60Pb40 alloy, 60 samples)

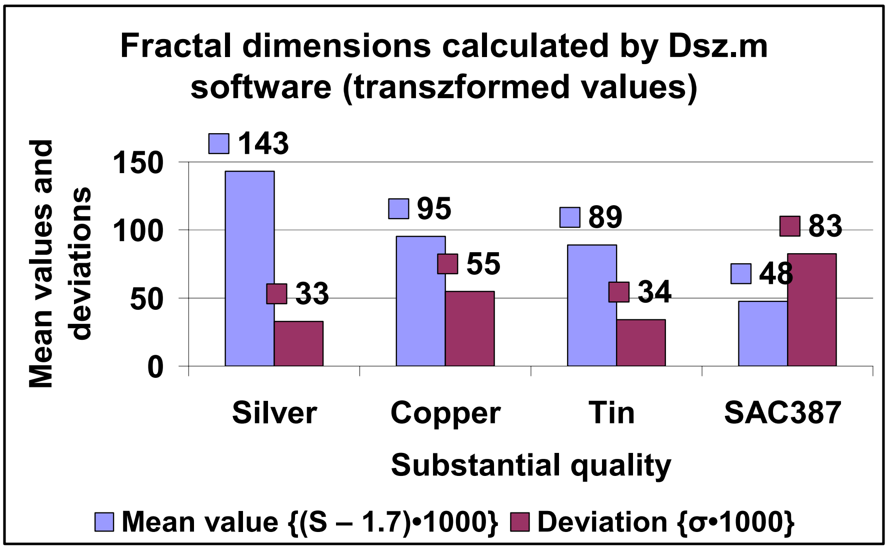


Fig. 11. Fractal dimensions calculated by Dsz.m software (transformed values).

spaced containers and calculates the number of elements in each container. We have tried to combine the curves with the histograms but resulted in crowded diagrams (Fig. 12). The best solution can be seen on Figs. 13-14 where the equation of curves is:

$$y = 100 \cdot (m\text{-}1.5) \cdot \exp(-\frac{1}{2} \cdot \frac{(x-m)^2}{s^2}) \tag{3}$$

The results of second and third experiment series show the same tendency. By the same program calculated fractal dimensions of various materials or alloys in every case representatively differ from each other, thus the uniformly determined fractal dimensions can approximately well describe the chemical composition.

In the course of the statistical evaluation and analysis of fractal dimensions we have observed a second relationship: the fractal dimension of an alloy is determined by the fractal dimensions of its compounds. The logarithm of the dendrites fractal dimension of an alloy (S) is nearly equal to the linear combination of logarithms of its components characteristic fractal dimensions (S_i) weighted by their molar ratio (n_i). Thus the equations will be formed as follows:

$$\log_a(S) = \sum_{i=1}^{k} n_i \cdot \log_a(S_i) \tag{4}$$

An important notice can be taken - eutectic alloys must be handled as a special material! Examples are shown in Tab. 2.

4 Conclusion

In this paper, the fractal description of dendrites formed in electrochemical migration processes and its usability in failure analysis was discussed. The most common materials were examined which are applied as solder materials or surface finishes

Fig. 12. Fractal dimensions of dendrites grown from SAC387 alloy and copper.

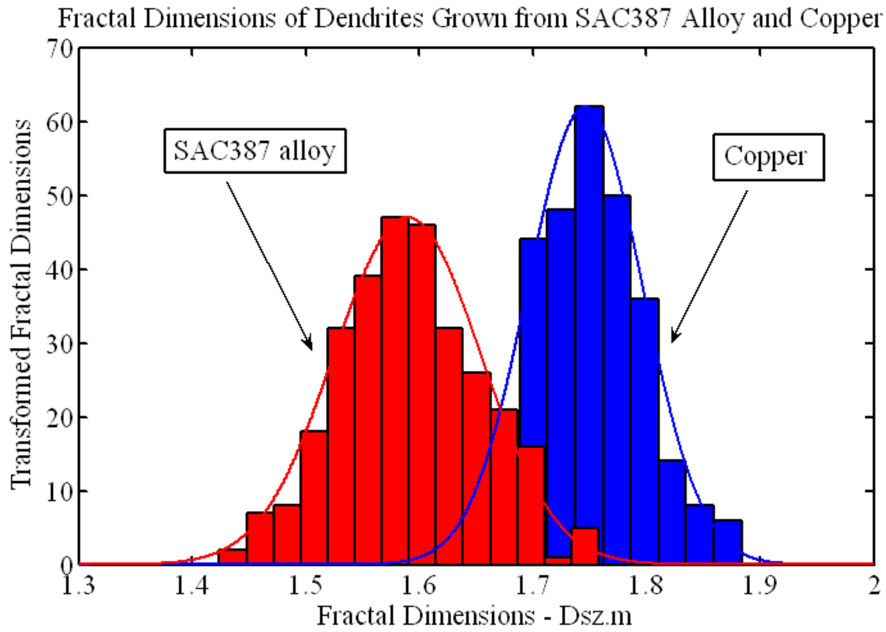


Fig. 13. Fractal dimensions of dendrites grown from silver, copper, tin and SAC387 alloy (Calculated by Dsz.m software, transformed values).

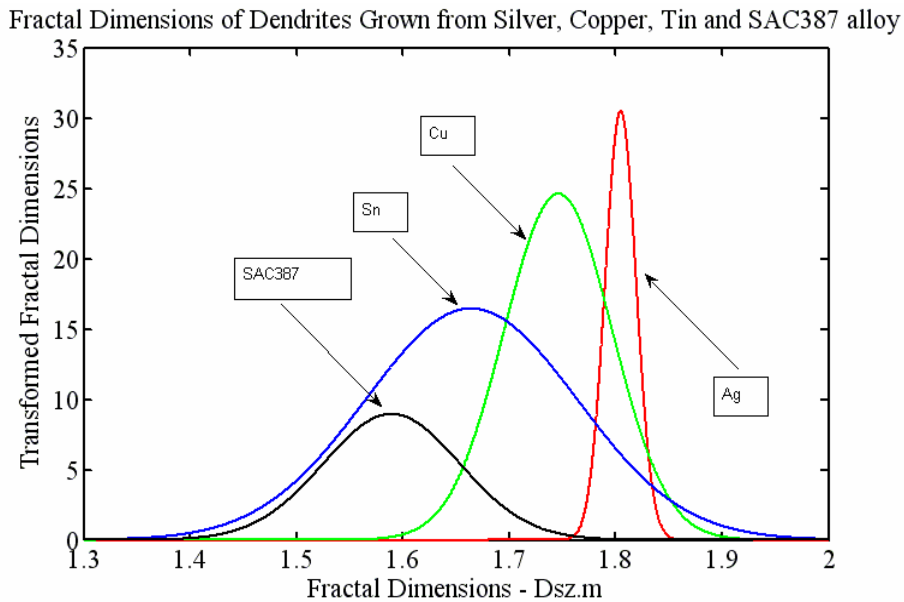
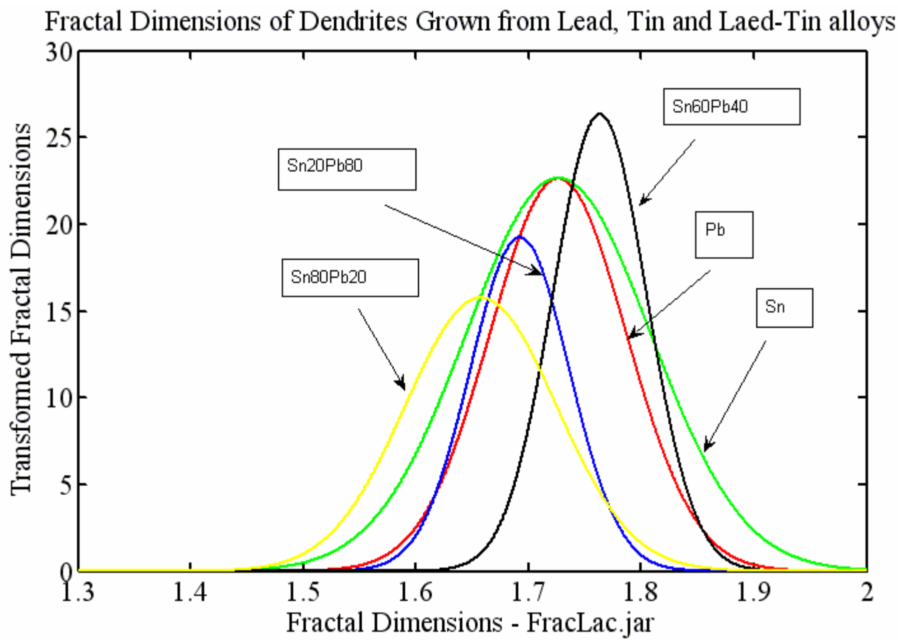


Fig. 14. Fractal dimensions of dendrites grown from lead, tin and lead-tin alloys (Calculated by ImageJ - FracLac.jar, transformed values).



Tab. 2. Comparison of mean value and calculated value of SAC387 alloy's fractal dimension (three examples)

Softwares	Mean value of dimensions	Theoretically calculated dimension values
Dff.m	1,76	1,74 ± 0,03
Dsz.m	1,79	1,75 ± 0,04
FracLac.jar	1,53	1,46 ± 0,07

in PWB manufacturing and assembling: tin, copper, silver and lead, as well as their alloys in solder materials – both conventional tin-lead alloys and lead-free solders like SAC (tin-silver-copper) were examined. Dendrites were grown artificially using systematically designed water drop test (WDT) series to get reproducible specimens with structures enabling examinations with sufficient accuracy. After taking optical photomicrographs, using image processing and computerized algorithms we have got fractal dimension values defined by several ways. The most important results can be summarized as follows:

1 We have proved our hypothesis that fractal dimensions of dendrites are parameters characteristic for their material composition both in the case of pure elements and in the case of alloys.

2 The careful evaluation of histograms demonstrated the good differentiability of fractal dimension values of dendrites formed from different metal elements. Although the values are dependent on the computerized algorithm of calculating fractal dimension, using the same method, they are characteristic to the material composition and can easily be differentiated from each other and calculated with simple methods.

3 In addition, we have found a simple equation for alloys: dendrites formed from alloys (containing all compounds of the original alloys) can be described with fractal dimension values that are molar weighted mean values of the fractal dimension values of the original components of the alloy. This may be a useful theoretical result for material sciences.

As an important practical aspect, fractal dimensions of dendrites formed during electrochemical migration processes can be used in failure analysis in "first and fast" identification of critical metallic elements without the application of chemical micro-analytical methods.

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